



Tutorials and worked examples for simulation,  
curve fitting, statistical analysis, and plotting.  
<http://www.simfit.org.uk>

The difference between 1-way ANOVA and 2-way ANOVA is that, whereas 1-way ANOVA only tests for differences between column means, 2-way ANOVA also considers the possibility of effects dependent on the rows. Note that, because the number of observations is the same for each subject, the data can be input as a data matrix into the SIMFIT file selection control either from a data file, by typing in from the keyboard, or by copying and pasting from a spreadsheet.

### Worked example for treatments and clotting times

From the main SIMFIT menus choose [Statistics] followed by [ANOVA] then select [2-way ANOVA] and, instead of using the default test file `anova2.tf1`, use the [Browse] feature on the file selection control to search for and then open the SIMFIT test file `anova2.tf2` which has the following data set.

Subject	Treatment			
	1	2	3	4
1	8.40	9.40	9.80	12.2
2	12.8	15.2	12.9	14.4
3	9.60	9.10	11.2	9.80
4	9.80	8.80	9.90	12.0
5	8.40	8.20	8.50	8.50
6	8.60	9.90	9.80	10.9
7	8.90	9.00	9.20	10.4
8	7.90	8.10	8.20	10.0

These are clotting times in minutes from eight subjects treated by four methods and analysis leads to the following results, indicating both subject and treatment dependent effects.

2-Way Analysis of Variance: Grand mean 9.994					
Source	SSQ	NDOF	MSSQ	F	p
Between rows (Subjects)	78.99	7	11.28	17.20	0.0000
Between columns (Treatments)	13.02	3	4.339	6.615	0.0025
Residual	13.77	21	0.6559		
Total	105.8	31			

Note that now there are variance ratio test statistics  $F$  and corresponding  $p$  values for both the rows and the columns and the calculations involved in constructing this ANOVA 2-way table follow.

### The assumed linear model

The 2-way ANOVA procedure is used when you want to include row and column effects in a completely randomized design, i.e., assuming no interaction and one replicate per cell so that the appropriate linear model is

$$y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}$$

$$\sum_{i=1}^r \alpha_i = 0$$

$$\sum_{j=1}^c \beta_j = 0$$

for a data matrix with  $r$  rows and  $c$  columns, i.e.  $n = rc$ .

### Calculating the variance ratio statistics

The mean sums of squares and degrees of freedom for row and column effects are worked out, then the appropriate  $F$  and  $p$  values are calculated. Using  $R_i$  for the row sums,  $C_j$  for the column sums, and  $T = \sum_{i=1}^r R_i = \sum_{j=1}^c C_j$  for the sum of observations, these are

$$\text{Row } SSQ = \sum_{i=1}^r R_i^2/c - T^2/n, \text{ with } DF = r - 1$$

$$\text{Column } SSQ = \sum_{j=1}^c C_j^2/r - T^2/n, \text{ with } DF = c - 1$$

$$\text{Total } SSQ = \sum_{i=1}^r \sum_{j=1}^c y_{ij}^2 - T^2/n, \text{ with } DF = n - 1$$

$$\text{Residual } SSQ = \text{Total } SSQ - \text{Row } SSQ - \text{Column } SSQ, \text{ with } DF = (r - 1)(c - 1)$$

where Row  $SSQ$  is the between rows sums of squares, Column  $SSQ$  is the between columns sum of squares, Total  $SSQ$  is the total sum of squares and Residual  $SSQ$  is the residual, or error sum of squares. Now two  $F$  statistics can be calculated from the mean sums of squares as

$$F_R = \frac{\text{Rows } MS}{\text{Residual } MS}$$

$$F_C = \frac{\text{Column } MS}{\text{Residual } MS}$$

The statistic  $F_R$  is compared with  $F(r - 1, (r - 1)(c - 1))$  to test

$$H_R : \alpha_i = 0, i = 1, 2, \dots, r$$

i.e., absence of row effects, while  $F_C$  is compared with  $F(c - 1, (r - 1)(c - 1))$  to test

$$H_C : \beta_j = 0, j = 1, 2, \dots, c$$

i.e., absence of column effects.

As with 1-way ANOVA, normality is assumed, and the technique can be extended to the case with replicates if it is wished to study variation within cells.