Random walks can be used to model numerous natural processes such as diffusion, or the migration of motile cells in a permeable gel matrix

The SɪᴍFɪT program **rannum** provides numerous ways to generate random walks with the following options.

1. There can be 1, 2, or 3 dimensions starting from user-defined initial coordinates.

2. The distributions used for the variables can be chosen independently for each dimension.

3. Simulated walks can be displayed, tabulated, or written to files.

4. The walks can be disjoint where each coordinate is varied in sequence, or coincident when all variables change at the same step.

Example 1: Disjoint or coincident walks

To illustrate the distinction between disjoint and coincident walks consider the next two examples.
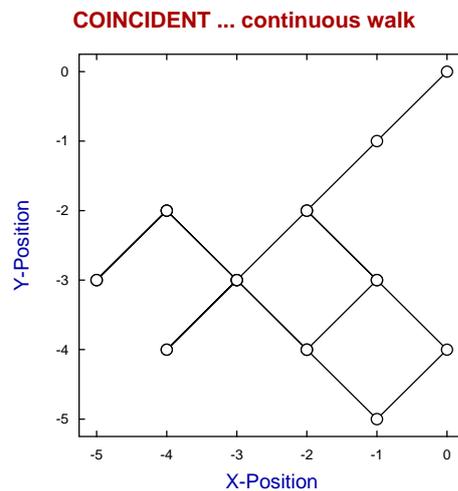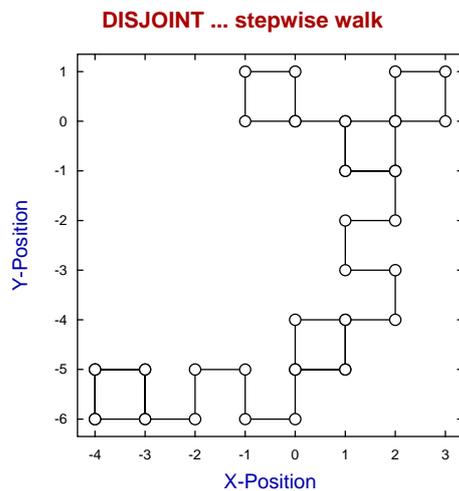
The distribution chosen for the two variables X and Y was a random integer with just two possible values, that is

$$P(X = -1) = p$$
$$P(X = +1) = 1 - p$$
$$P(Y = -1) = p$$
$$P(Y = +1) = 1 - p$$
$$p = \frac{1}{2}.$$

In the disjoint mode the walk takes place first along the X coordinate then along the Y coordinate, so that all movements can clearly be seen to be either horizontal or vertical, whereas in the coincident mode the walk on the grid occurs in diagonal jumps from node to node. Tables and data written to file are in the more economical coincident mode.

Example 2: Plotting collections of walks

It is often useful to plot several walks starting from the same position as it is often surprising how such repeated walks diverge. The examples below illustrate a common phenomenon where walks may tend to persist in the same direction even with equiprobable positive and negative steps, and which can appear somewhat contrary to intuition. The examples illustrated all had the probability of a step length of 1 equal to the probability of a step length of -1.

Random Walk in One Dimension
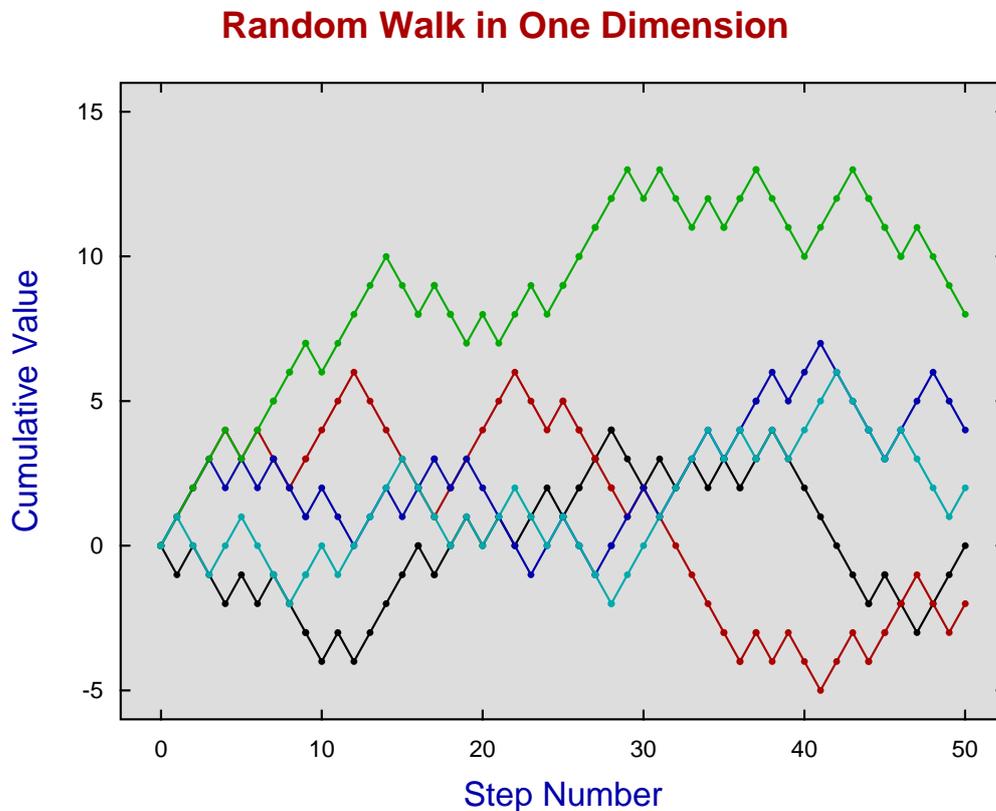
There are two ways to create such plots depending on the technique used to archive the coordinates.

- Using the [Advanced] option

  When it is a wished to archive the coordinates of a walk then the [Advanced] button should be used to provide the option to save the coordinates to a file. After the file has been saved you will have the additional option to store the file in your graphics project archive. This is simply a very convenient technique to store graphics files for subsequent re-plotting. There are then three ways to create a graph with the collected walks using program **simfit**.

  1. Select the files individually.
  2. Select the required files as a set from a library file. created by program **maklib**.
  3. Select the required files by multiple file selection from your project archive.

- Using coordinates written to a file

  Files of random walks saved from program **rannum** can be used directly for retrospective plotting as, with the special case of just one dimension, the step number is added as a first column to the file so it can be used as the *X* coordinate.

2

Example 3: Plotting two dimensional random walks

The previous examples used steps of 1 or -1 with equal probability but other distributions are often much more useful to represent natural processes.

For example, the next example illustrates five random walks all starting at $x = 0$ and $y = 0$ to illustrate the large variation in walks possible where the steps were all taken from a standard normal distribution.

Random Walk in Two Dimensions
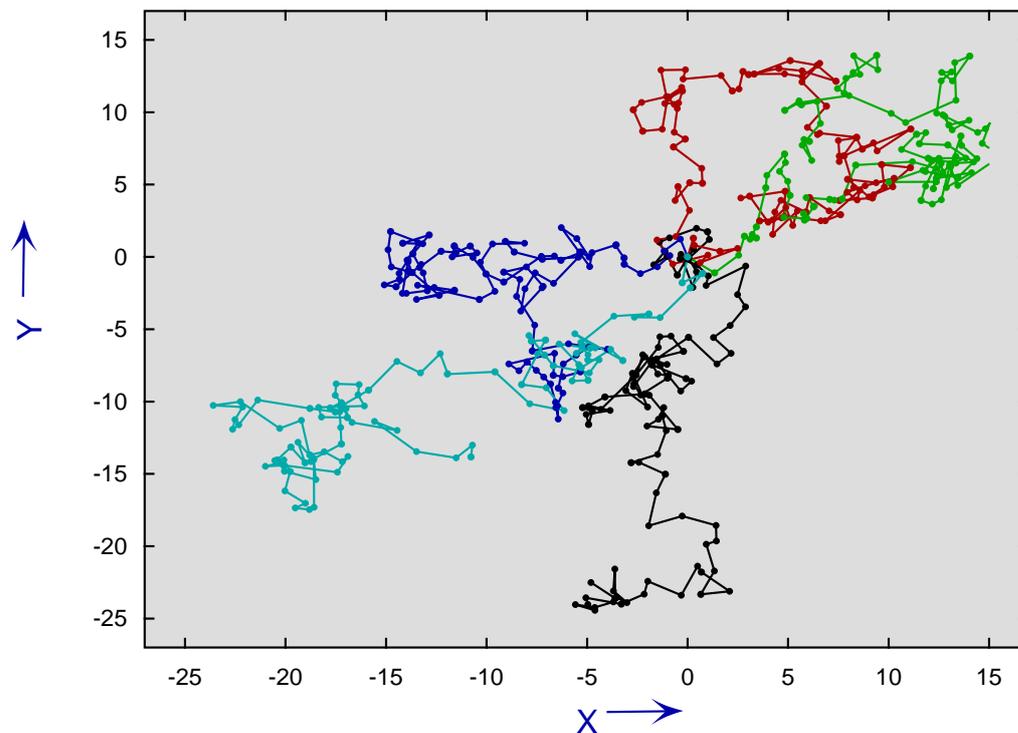
In random walks where equally probable steps of -1 or 1 are not used then the walk is no longer taking place between the integer intersection points on rectangular grids, instead jumps can be taken between arbitrary coordinates as allowed by the distributions selected. In the previous graph the distribution used for the steps x_i, y_i has density function

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2$$

with $\mu = 0$ and $\sigma = 1$, and so jumps larger than 3 or smaller than -3 are not very likely.

To summarize: after n random steps x_i, y_i the coordinates X_n, Y_n plotted would be

$$X_n = X_{start} + \sum_{i=1}^{n} x_i$$

$$Y_n = Y_{start} + \sum_{i=1}^{n} y_i.$$

Example 4: Plotting three dimensional random walks

Three dimensional random walks are no different than two dimensional walks except that the procedure for plotting collections of walks differs somewhat.

For instance, the next graph shows a typical three dimensional random walk from the starting point

$$x = 0, \ y = 0, \ z = 0$$

where a standard normal distribution was used for all three variables.

Random Walk in Three Dimensions
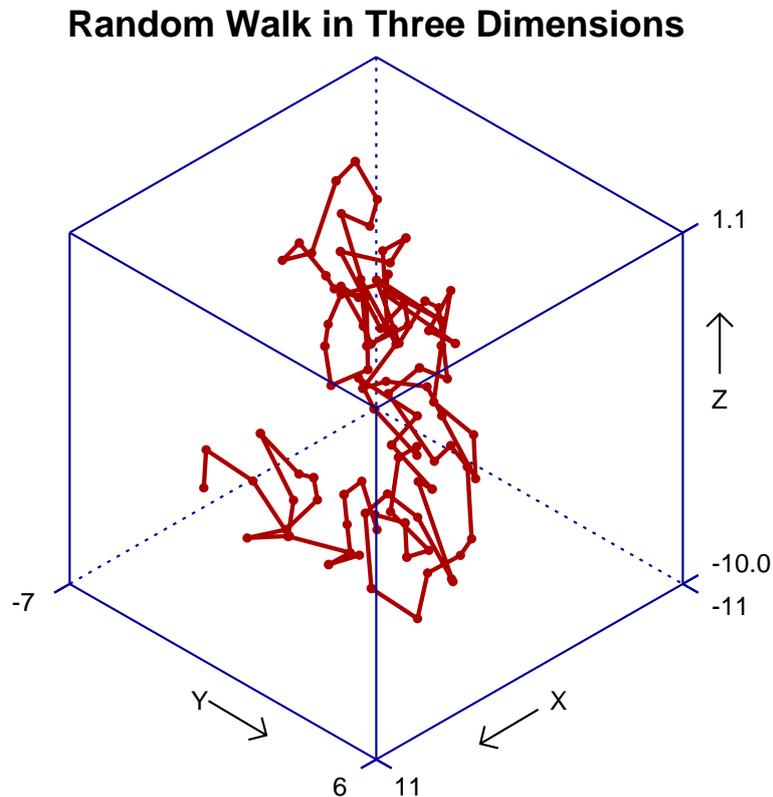
To plot collections of such walks, the walks are first saved from program **rannum** to coordinate files. Then program **simplot** is opened and the coordinate files are supplied using the option for plotting space curves. This anticipates data in parametric form. In other words, it is assumed that the three columns in the coordinate files represent the triples

$$x = f(t)$$
$$y = g(t)$$
$$z = h(t)$$

for arbitrary functions $f(.), g(.),$ and $h(.)$ evaluated at equal increments of parameter t.

Note that this option should not be confused with the option to plot three dimensional surfaces.